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Two collinear Griffith cracks subjected to uniform tension in infinitely long strip

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Abstract

The problem of determining the stress field in an elastic strip of finite width when the uniform tension is applied to the faces of two collinear symmetrical cracks situated within it is considered. By using the Fourier transform, the problem can be solved with a set of triple integral equations. These equations are solved using Schmidt's method. This method is suitable for solving the strip's problem of arbitrary width.

1. Introduction

From the engineering and the fracture mechanics point of view, the crack problems of the strip are of particular interest. For this reason, the strip problems were treated by many researchers using different methods. For example Irwin (1957), Sneddon and Srivastav (1971), Gupta (1973), Lowengrub (1966), and Lowengrub and Srivastava (1968b). Irwin (1957) gave an approximation by periodic crack solution to determine the distribution of the stress in the neighborhood of two collinear edge cracks. The stress field of a Griffith crack in a strip of finite width has been discussed by Sneddon (1971) and Gupta (1973) using the integral transform method. The problem of the crack parallel to the edges of the strip had been treated by Lowengrub (1966) and Lowengrub and Srivastava (1968b) by using the integral transform method of a long time ago. However, Lowengrub's (1966) and Lowengrub and Srivastava (1968b) solution is only valid for $h \gg 1$ (h is the half width of the strip), and is not a closed form solution. It is only an approximate solution. The results of the small strip widths ($h < 1$) cannot be found in the published papers.

In the present paper, the same problem which was treated by Lowengrub and Srivastava (1968b) is reworked using a somewhat different method, namely Schmidt's method (see e.g. Morse and Feshbach, 1959; Itou, 1980). It is important that this method is suitable for solving the strip's

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problem of arbitrary width, and it is simple and convenient for solving this problem. The Schmidt's method has been used to solve many problems of fracture mechanics (see e.g. Itou, 1978a, b, 1979, 1980). It can be seen that the Schmidt's method is performed satisfactorily to the demand of the precision. The Fourier transform is applied and a mixed boundary value problem is reduced to a set of triple integral equations. In solving the triple integral equations, the crack surface displacement is expanded in a series using Jacobi's polynomials and Schmidt's method (see e.g. Morse and Feshbach, 1958; Itou, 1980) is used. This process is quite different from that adopted in references (see e.g. Irwin, 1957; Sneddon and Srivastav, 1971; Gupta, 1973; Lowengrub, 1966; Lowengrub and Srivastava 1968b).

2. Formulation of the problem

Consider an infinitely long, homogeneous, isotropic, elastic strip of width $2h$, containing two collinear symmetrical cracks parallel to the edges of the strip. These cracks occupy the intervals $-1 \leqslant x \leqslant -a$, $a \leqslant x \leqslant 1$, $y = 0$. The geometry of the problem is shown in Fig. 1. Let the components of the displacement in x-, y-, z-directions be given by u, v and w. For this problem, w vanishes everywhere, u and v only are functions of coordinates x, y. [The solution of the strip of width $2h$ containing two Griffith edge cracks of length b–a can easily be obtained by a simple change in the numerical values of the present paper ($b > a > 0$).]

The corresponding stress field consists of three stresses

$$\sigma_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} \tag{1}$$

$$\sigma_{yy} = (\lambda+2\mu)\frac{\partial v}{\partial y} + \lambda\frac{\partial u}{\partial x} \tag{2}$$

$$\sigma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{3}$$

while all other components vanish. In eqns (1)–(3), λ and μ are the classical Lamé constants.

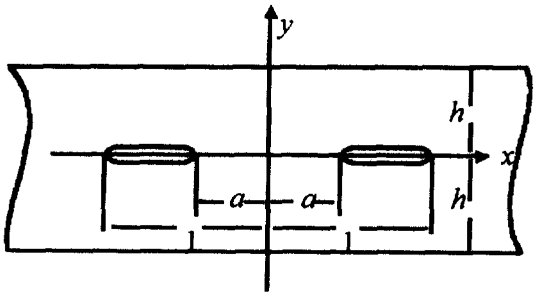

Fig. 1. An elastic strip containing two collinear cracks.

Substituting eqns (1)–(3) into the equation of elasticity, the displacement equations can be obtained as follows:

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\,\partial y}=0 \tag{4}$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+\mu\frac{\partial^2 v}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\,\partial y}=0 \tag{5}$$

The boundary conditions are

$$\sigma_{xy}(x,0)=0,\quad \sigma_{yy}(x,0)=-\tau_0,\quad a\leqslant|x|\leqslant 1 \tag{6}$$

$$\sigma_{xy}(x,0)=0,\quad v(x,0)=0,\quad 0<|x|<a,\quad 1<|x| \tag{7}$$

$$\sigma_{xy}(x,\pm h)=\sigma_{yy}(x,\pm h)=0,\quad -\infty<x<\infty \tag{8}$$

$$u=v=0\quad \text{as } x\to\pm\infty \tag{9}$$

where τ_0 is a constant having the dimension of stress.

3. Analysis

From Lowengrub and Srivastava (1968b), it can be shown that suitable expressions for the appropriate stress and displacement components are as follows: ($y\geqslant 0$)

$$u=\frac{2}{\pi}\int_0^\infty A(s)\left\{U_1(s)\cosh(sy)+U_2(s)\left[\frac{\mu}{\lambda+\mu}\cosh(sy)+sy\sinh(sy)\right]\right\}\sin(sx)\,\mathrm{d}s$$
$$+\frac{2}{\pi}\int_0^\infty A(s)\left[\frac{\mu}{2(\lambda+\mu)}\sinh(sy)+\frac{1}{2}sy\cosh(sy)\right]\sin(sx)\,\mathrm{d}s \tag{10}$$

$$v=\frac{2}{\pi}\int_0^\infty A(s)\left\{U_1(s)\sinh(sy)+U_2(s)\left[\frac{\lambda+2\mu}{\lambda+\mu}\sinh(sy)+sy\cosh(sy)\right]\right\}\cos(sx)\,\mathrm{d}s$$
$$+\frac{2}{\pi}\int_0^\infty A(s)\left[-\frac{\lambda+2\mu}{2(\lambda+\mu)}\cosh(sy)-\frac{1}{2}sy\sinh(sy)\right]\cos(sx)\,\mathrm{d}s \tag{11}$$

$$\sigma_{yy}=\frac{4\mu}{\pi}\int_0^\infty sA(s)\{U_1(s)\cosh(sy)-U_2(s)[\cosh(sy)-sy\sinh(sy)]\}\cos(sx)\,\mathrm{d}s$$
$$+\frac{2\mu}{\pi}\int_0^\infty sA(s)[\sinh(sy)-sy\cosh(sy)]\cos(sx)\,\mathrm{d}s \tag{12}$$

$$\sigma_{xy} = \frac{4\mu}{\pi}\int_0^\infty sA(s)\{-U_1(s)\sinh(sy) - U_2(s)sy\cosh(sy)\}\sin(sx)\,ds + \frac{2\mu}{\pi}\int_0^\infty sA(s)[sy\sinh(sy)]\sin(sx)\,ds \quad (13)$$

where

$$U_1(s) = (hs)^2/[2hs+\sinh(2hs)]$$
$$U_2(s) = \sinh^2(hs)/[2hs+\sinh(2hs)]$$

Because of symmetry, it suffices to consider the problem in the first quadrant only. It is a simple matter to verify that these expressions fulfil the conditions (6)–(9). The conditions (6)–(9) will be satisfied if we can find a function $A(s)$ satisfying a set of triple integral equations:

$$\int_0^\infty A(s)\cos(sx)\,ds = 0, \quad 0 < x < a, \quad 1 < x \quad (14)$$

$$\int_0^\infty sA(s)[1+M(hs)]\cos(sx)\,ds = \frac{\pi\tau_0}{2\mu}, \quad a \leqslant x \leqslant 1, \quad (15)$$

where

$$M(u) = \frac{-2u(u+1)+e^{-2u}-1}{2u+\sinh(2u)} \quad (16)$$

To solve integral eqns (14) and (15), the displacement v can be represented by the following series: (any function can be approached by using a complete orthonormal function sequence series).

$$v(x,0) = \sum_{n=0}^{\infty} a_n P_n^{(1/2,1/2)}\left(\frac{|x|-\frac{1+a}{2}}{\frac{1-a}{2}}\right)\left(1-\frac{\left(|x|-\frac{1+a}{2}\right)^2}{\left(\frac{1-a}{2}\right)^2}\right)^{1/2}, \quad \text{for } a \leqslant |x| \leqslant 1$$
$$= 0, \quad \text{for } 0 < |x| < a, \quad 1 < x, \quad (17)$$

where a_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial (see e.g. Gradshteyn and Ryzhik, 1980). The Fourier cosine transform for eqn (17) is (see e.g. Erdelyi, 1954)

$$\frac{\lambda+2\mu}{2(\lambda+\mu)}A(s) = \bar{v}(s,0) = \sum_{n=0}^{\infty} a_n G_n(s) B_n J_{n+1}\left[s\left(\frac{1-a}{2}\right)\right]s^{-1}, \quad (18)$$

in which

$$G_n(s) = \begin{cases} (-1)^{n/2}\cos\left(s\dfrac{1+a}{2}\right), & n = 0,2,4,6,\ldots \\ (-1)^{(n-1)/2}\sin\left(s\dfrac{1+a}{2}\right), & n = 1,3,5,7,\ldots \end{cases} \qquad B_n = 2\sqrt{\pi}\frac{\Gamma\left(n+1+\dfrac{1}{2}\right)}{n!}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substitution of eqn (18) into eqns (14) and (15), respectively, eqn (14) has been automatically satisfied by using the Fourier transform. Then the remaining eqn (15) reduces to the form after integration with respect to x in $a \leqslant x \leqslant 1$

$$\sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} s^{-1} G_n(s) J_{n+1}\left(\frac{1-a}{2}s\right)[1+M(hs)][\sin(sx)-\sin(sa)]\,\mathrm{d}s = \frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}(x-a) \qquad (19)$$

The semi-infinite integral in eqn (19) can be modified as (see e.g. Gradshteyn and Ryzhik, 1980)

$$\int_0^{\infty} \frac{1}{s} J_{n+1}\left(s\frac{1-a}{2}\right)[1+M(hs)]\cos\left(s\frac{1+a}{2}\right)\sin(sx)\,\mathrm{d}s$$

$$= \frac{1}{2(n+1)}\left\{ \frac{\left(\dfrac{1-a}{2}\right)^{n+1}\sin\left(\dfrac{(n+1)\pi}{2}\right)}{\left\{x+\dfrac{1+a}{2}+\sqrt{\left(x+\dfrac{1+a}{2}\right)^2-\left(\dfrac{1-a}{2}\right)^2}\right\}^{n+1}} - \sin\left[(n+1)\sin^{-1}\left(\frac{1+a-2x}{1-a}\right)\right]\right\}$$

$$+\int_0^{\infty}\frac{1}{s}J_{n+1}\left(s\frac{1-a}{2}\right)M(hs)\cos\left(s\frac{1+a}{2}\right)\sin(sx)\,\mathrm{d}s, \qquad (20)$$

$$\int_0^{\infty} \frac{1}{s} J_{n+1}\left(s\frac{1-a}{2}\right)[1+M(hs)]\sin\left(s\frac{1+a}{2}\right)\sin(sx)\,\mathrm{d}s$$

$$= \frac{1}{2(n+1)}\left\{ \cos\left[(n+1)\sin^{-1}\left(\frac{1+a-2x}{1-a}\right)\right]\right.$$

$$
\left. - \frac{\left(\frac{1-a}{2}\right)^{n+1} \cos\left(\frac{(n+1)\pi}{2}\right)}{\left\{x+\frac{1+a}{2}+\sqrt{\left(x+\frac{1+a}{2}\right)^2-\left(\frac{1-a}{2}\right)^2}\right\}^{n+1}} \right\}
+\int_0^{\infty} \frac{1}{s} J_{n+1}\left(s\frac{1-a}{2}\right) M(hs) \sin\left(s\frac{1+a}{2}\right) \sin(sx)\,\mathrm{d}s \tag{21}
$$

The semi-infinite integrals on the right of eqns (20) and (21) can be evaluated numerically by Filon's method (see e.g. Amemiya and Taguchi, 1969). Thus, eqn (19) can be solved for coefficients a_n by the Schmidt method (see e.g. Morse and Feshbach, 1958; Itou, 1980). For brevity, eqn (19) can be written as

$$
\sum_{n=0}^{\infty} a_n E_n(x) = U(x), \quad a \leqslant x \leqslant 1 \tag{22}
$$

where $E_n(x)$ and $U(x)$ are known functions and coefficients a_n are unknown and will be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$
\int_a^1 P_m(x) P_n(x)\,\mathrm{d}x = N_n \delta_{mn}, \quad N_n = \int_a^1 P_n^2(x)\,\mathrm{d}x \tag{23}
$$

can be constructed from the function, $E_n(x)$, such that

$$
P_n(x) = \sum_{i=0}^{n} \frac{M_{in}}{M_{nn}} E_i(x) \tag{24}
$$

where M_{in} is the cofactor of the element d_{in} of D_n, which is defined as

$$
D_n = \begin{bmatrix} d_{00}, d_{01}, d_{02}, \ldots, d_{0n} \\ d_{10}, d_{11}, d_{12}, \ldots, d_{1n} \\ d_{20}, d_{21}, d_{22}, \ldots, d_{2n} \\ \cdots \\ \cdots \\ \cdots \\ d_{n0}, d_{n1}, d_{n2}, \ldots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_a^1 E_i(x) E_j(x)\,\mathrm{d}x \tag{25}
$$

Using eqns (22)–(25), it can be obtained

$$
a_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \tag{26}
$$

with

$$q_j = \frac{1}{N_j}\int_a^1 U(x)P_j(x)\,dx \tag{27}$$

4. Stress intensity factor

When coefficients a_n are known, the entire stress field is obtainable. However, in fracture mechanics, it is of importance to determine stress σ_{yy} in the vicinity of the crack's tips. σ_{yy} at $y = 0$ is given by

$$\sigma_{yy} = -\frac{4\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} G_n(s)[1+M(hs)]J_{n+1}\left(s\frac{1-a}{2}\right)\cos(sx)\,ds \tag{28}$$

From the fracture mechanics point of view, only the singular stress near the crack tip will be obtained here. Observing the expression in eqn (28), the singular portion of the stress field results can be obtained from the relationship (see e.g. Gradshteyn and Ryzhik, 1980)

$$\cos\left(s\frac{1+a}{2}\right)\cos(sx) = \frac{1}{2}\left\{\cos\left[s\left(\frac{1+a}{2}-x\right)\right]+\cos\left[s\left(\frac{1+a}{2}+x\right)\right]\right\}$$

$$\sin\left(s\frac{1+a}{2}\right)\cos(sx) = \frac{1}{2}\left\{\sin\left[s\left(\frac{1+a}{2}-x\right)\right]+\sin\left[s\left(\frac{1+a}{2}+x\right)\right]\right\}$$

$$\int_0^{\infty} J_n(sa)\cos(bs)\,ds = \begin{cases} \dfrac{\cos[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a>b \\ -\dfrac{a^n\sin(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b>a \end{cases}$$

$$\int_0^{\infty} J_n(sa)\sin(bs)\,ds = \begin{cases} \dfrac{\sin[n\sin^{-1}(b/a)]}{\sqrt{a^2-b^2}}, & a>b \\ \dfrac{a^n\cos(n\pi/2)}{\sqrt{b^2-a^2}[b+\sqrt{b^2-a^2}]^n}, & b>a \end{cases}$$

The singular portion of the stress field can be expressed as follows

$$\sigma = -\frac{2\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\sum_{n=0}^{\infty} a_n B_n H_n(a,x) \tag{29}$$

where

$$H_n(a,x) = -F_1(a,x,n),\quad n = 0,1,2,3,4,5,\ldots,\quad (\text{for } 0<x<a)$$

$$H_n(a,x) = \begin{cases} -F_2(a,x,n), & n = 0,2,4,6,\ldots \\ F_2(a,x,n), & n = 1,3,5,7,\ldots \end{cases},\quad (\text{for } 1<x)$$

$$F_1(a,x,n) = \frac{2(1-a)^{n+1}}{\sqrt{(1+a-2x)^2-(1-a)^2}[1+a-2x+\sqrt{(1+a-2x)^2-(1-a)^2}]^{n+1}}$$

$$F_2(a,x,n) = \frac{2(1-a)^{n+1}}{\sqrt{(2x-1-a)^2-(1-a)^2}[2x-1-a+\sqrt{(2x-1-a)^2-(1-a)^2}]^{n+1}}$$

At the left end of the right crack, the stress intensity factor K_{L1} can be obtained as

$$K_{L1} = \lim_{x\to a^-}\sqrt{2\pi(a-x)}\cdot\sigma = \frac{4\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\sqrt{\frac{\pi}{2(1-a)}}\sum_{n=0}^{\infty}a_nB_n \tag{30}$$

At the right end of the right crack, the stress intensity factor K_{R1} can be obtained as

$$K_{R1} = \lim_{x\to 1^+}\sqrt{2\pi(x-1)}\cdot\sigma = \frac{4\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\sqrt{\frac{\pi}{2(1-a)}}\sum_{n=0}^{\infty}a_nB_n(-1)^n \tag{31}$$

5. Numerical example and results

The dimensionless stress intensity factors K_{L1} and K_{R1} are carried out numerically. Adopting the first ten terms of the infinite series to eqn (22), we performed the Schmidt procedure. For a check of the accuracy, the values of $\Sigma_{n=0}^{9} a_nE_n(x)$ and $U(x)$ are given in Table 1 for $a = 0.5$, $h = 1.5$. In Table 2, the values of the coefficients a_n are given for $a = 0.5$, $h = 1.5$. From the above results and references (see e.g. Itou, 1978a, b, 1979, 1980), it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series to eqn (22) are obtained. The behavior of the solution stays steady with an increase of the number of terms in eqn (22). Hence, it is clear that the Schmidt's method is carried out satisfactorily. The precision of present paper's solution can satisfy the demands of the practical problem. In all computation, the material constants are

Table 1
Values of

$$\sum_{n=0}^{9} a_nE_n(x)\bigg/\left(\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right) \quad \text{and} \quad U(x)\bigg/\left(\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right)$$

for $a = 0.5$, $h = 1.5$

x	$\sum_{n=0}^{9} a_nE_n(x)\bigg/\left(\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right)$	$U(x)\bigg/\left(\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right) = x-a$
0.5	0.0000	0.0000
0.6	0.1001	0.1000
0.7	0.1998	0.2000
0.8	0.2997	0.3000
0.9	0.4001	0.4000

Table 2
Values of

$$a_n \Big/ \left(\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right)$$

for $a = 0.5$, $h = 1.5$

n	$a_n \Big/ \left(\frac{\pi\tau_0(\lambda+2\mu)}{4\mu(\lambda+\mu)}\right)$
0	0.167704E+00
1	−0.494209E−03
2	−0.303280E−04
3	−0.457772E−05
4	0.850114E−06
5	−0.147793E−07
6	−0.109022E−08
7	−0.260762E−09
8	0.392335E−10
9	−0.231649E−11

not considered because the stress intensity factors do not depend on the material constants. It only depends on the crack length and the width of the strip. All finite and semi-infinite integrals are evaluated with Simpson's and Filon's methods, respectively. The solution of the strip of width $2h$ containing two Griffith edge cracks of arbitrary length b–a can easily be obtained by a simple change in the numerical values of the present paper ($b > a > 0$), i.e. it can use the results of the strip of half width h/b containing two Griffith cracks of length $1-a/b$ in the present paper. The results of this paper are suitable for the strip of arbitrary width. However, the solution of Lowengrub and Srivastava's (1968b) is only suitable for $h \gg 1$, and it is only an approximate solution. The variations with a and h of the dimensionless stress intensity factors $K_{L\text{I}}$ and $K_{R\text{I}}$ of the present paper are given in Tables 3–9. The results of the dimensionless stress intensity factors $K_{L\text{I}}$ and $K_{R\text{I}}$ become higher with the increase of the crack length, and the stress intensity factors become lower with the increase of the strip width. For small strip widths ($h \leqslant 1$), the stress intensity factors at the inner crack tips are smaller than ones at the outer crack tips for $0.5 \geqslant a \geqslant 0.1$ in Table 3, $0.9 \geqslant a \geqslant 0.2$ in Table 4, $0.9 \geqslant a \geqslant 0.5$ in Table 5 and $0.9 \geqslant a \geqslant 0.8$ in Table 6, respectively. The results of the small strip width ($h \leqslant 1$) cannot be found in published papers. This kind of phenomenon should be further investigated. In Tables 7 and 8, for the cases $h = 1.2$, $h = 3.0$, the results in the present paper are different from Lowengrub's (see e.g. Lowengrub and Srivastava, 1968b). It can be shown that the solution of Lowengrub and Srivastava's (1968b) is not suitable for the case $h \leqslant 3.0$. For the case of $h = 5.0$, in Table 9, the results in this paper show a similar tendency to ones obtained by Lowengrub and Srivastava (1968b) for the strip problem and by Lowengrub and Srivastava (1968a) for two collinear cracks in an infinite medium. The solution of Lowengrub and Srivastava's (1968a) is an exact one for two collinear cracks in an infinite medium. From the

Table 3
Variation with a of the intensity factors K_{L1} and K_{R1} for $h = 0.3$

a	K_{L1}/τ_0	K_{R1}/τ_0
0.1	2.44596	2.75753
0.2	2.28021	2.47127
0.3	2.07466	2.12158
0.4	1.75516	1.76175
0.5	1.42654	1.42690
0.6	1.12636	1.12629
0.7	0.861032	0.861011
0.8	0.627783	0.627781
0.9	0.408723	0.408723

Table 4
Variation with a of the intensity factors K_{L1} and K_{R1} for $h = 0.5$

a	K_{L1}/τ_0	K_{R1}/τ_0
0.1	2.12466	1.88188
0.2	1.65257	1.67217
0.3	1.44443	1.48364
0.4	1.27218	1.29357
0.5	1.09732	1.10538
0.6	0.922574	0.924887
0.7	0.752981	0.753472
0.8	0.585439	0.585505
0.9	0.400826	0.400829

Table 5
Variation with a of the intensity factors K_{L1} and K_{R1} for $h = 0.8$

a	K_{L1}/τ_0	K_{R1}/τ_0
0.1	1.99116	1.59034
0.2	1.50411	1.38420
0.3	1.26655	1.23177
0.4	1.10428	1.09715
0.5	0.968680	0.968849
0.6	0.840627	0.841647
0.7	0.710586	0.711108
0.8	0.569543	0.569678
0.9	0.397976	0.397987

Table 6
Variation with a of the intensity factors K_{L1} and K_{R1} for $h = 1.0$

a	K_{L1}/τ_0	K_{R1}/τ_0
0.1	1.91068	1.53198
0.2	1.46489	1.33154
0.3	1.23655	1.18520
0.4	1.07781	1.05924
0.5	0.946927	0.941157
0.6	0.825377	0.824012
0.7	0.702011	0.701831
0.8	0.566122	0.566129
0.9	0.397335	0.397338

Table 7
The variation with a of the intensity factors K_{L1} and K_{R1} for $h = 1.2$ and compared with the results K^1_{L1}, K^1_{R1} of Lowengrub and Srivastava's (1968b)

a	K_{L1}/τ_0	K_{R1}/τ_0	K^1_{L1}/τ_0	K^1_{R1}/τ_0
0.1	1.83800	1.49679	1.12853	8.80854
0.2	1.43223	1.30583	1.05625	8.96689
0.3	1.21788	1.16459	1.24016	9.37041
0.4	1.06512	1.04303	1.68749	10.0077
0.5	0.937713	0.929345	2.50107	10.9284
0.6	0.819085	0.816378	3.87268	12.2506
0.7	0.698398	0.697722	6.17337	14.2395
0.8	0.564621	0.564516	10.2823	17.5975
0.9	0.397041	0.397035	19.4022	25.1382

Table 8
The variation with a of the intensity factors K_{L1} and K_{R1} for $h = 3.0$ and compared with the results K^1_{L1}, K^1_{R1} of Lowengrub and Srivastava's (1968b)

a	K_{L1}/τ_0	K_{R1}/τ_0	K^1_{L1}/τ_0	K^1_{R1}/τ_0
0.1	1.57333	1.35511	1.55833	1.54926
0.2	1.29591	1.22073	1.28915	1.42270
0.3	1.14143	1.11045	1.14461	1.32524
0.4	1.02175	1.00851	1.03991	1.24159
0.5	0.913545	0.908072	0.955322	1.16657
0.6	0.806301	0.804251	0.885693	1.09856
0.7	0.692380	0.691748	0.833246	1.03926
0.8	0.562436	0.562304	0.883238	1.03007
0.9	0.396645	0.396635	1.07857	1.18783

Table 9
The variation with a of the intensity factors K_{L1} and K_{R1} for $h = 5.0$ and compared with the results K^{1}_{L1}, K^{1}_{R1} of Lowengrub and Srivastava's (1968b) and compared with the results K^{*}_{L1}, K^{*}_{R1} of Lowengrub and Srivastava's (1968a)

a	K_{L1}/τ_0	K_{R1}/τ_0	K^{1}_{L1}/τ_0	K^{1}_{R1}/τ_0	K^{*}_{L1}/τ_0	K^{*}_{R1}/τ_0
0.02	2.42131	1.47158	2.42694	1.50273	2.36330	1.43788
0.05	1.84311	1.39992	1.84120	1.42578	1.79924	1.36791
0.1	1.52321	1.311734	1.52154	1.34291	1.49234	1.29165
0.2	1.26609	1.19621	1.26541	1.22270	1.14708	1.17893
0.3	1.12261	1.09441	1.12317	1.12249	1.11018	1.08274
0.4	1.01008	0.998327	1.01255	1.02872	1.00209	0.990705
0.5	0.906706	0.901983	0.912211	0.935615	0.901839	0.897294
0.6	0.802657	0.800941	0.813009	0.839166	0.799957	0.798319
0.7	0.690723	0.690210	0.709016	0.735293	0.689442	0.688957
0.8	0.561874	0.561770	0.594170	0.618343	0.561420	0.561323
0.9	0.396552	0.396545	0.459618	0.478641	0.396474	0.396467
0.95	0.280287	0.280286	0.383743	0.397902	0.280273	0.280272

above results, it can be shown that the solution of Lowengrub and Srivastava's (1968b) is only suitable for the large h. The solution of this paper is suitable for solving the strip's problem of arbitrary width. The results of this paper are approximate to ones of two collinear Griffith cracks in an infinite medium for the case of $h \geqslant 5.0$, that is the influence of the width of strip to the results is small for the case of $h \geqslant 5.0$. In Tables 3–9, the number of significant digits is six.

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